12. Using the floor as the reference position for computing potential energy, mechanical energy conservation leads to

$$\begin{aligned} U_{\text{release}} &= K_{\text{top}} + U_{\text{top}} \\ mgh &= \frac{1}{2}mv_{\text{com}}^2 + \frac{1}{2}I\omega^2 + mg(2R)\ . \end{aligned}$$

Substituting $I = \frac{2}{5}mr^2$ (Table 11-2(f)) and $\omega = v_{\text{com}}/r$ (Eq. 12-2), we obtain

$$\begin{aligned} mgh &= \frac{1}{2}mv_{\text{com}}^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{v_{\text{com}}}{r}\right)^2 + 2mgR \\ gh &= \frac{7}{10}v_{\text{com}}^2 + 2gR \end{aligned}$$

where we have canceled out mass m in that last step.

(a) To be on the verge of losing contact with the loop (at the top) means the normal force is vanishingly small. In this case, Newton's second law along the vertical direction ($+y$ downward) leads to

$$mg = ma_r \quad \Longrightarrow \quad g = \frac{v_{\text{com}}^2}{R-r}$$

where we have used Eq. 11-23 for the radial (centripetal) acceleration (of the center of mass, which at this moment is a distance $R-r$ from the center of the loop). Plugging the result $v_{\text{com}}^2 = g(R-r)$ into the previous expression stemming from energy considerations gives

$$gh = \frac{7}{10}(g)(R-r) + 2gR$$

which leads to

$$h = 2.7R - 0.7r \quad \approx \quad 2.7R\ .$$

(b) The energy considerations shown above (now with $h = 6R$) can be applied to point Q (which, however, is only at a height of R) yielding the condition

$$g(6R) = \frac{7}{10}v_{\text{com}}^2 + gR$$

which gives us $v_{\text{com}}^2 = 50gR/7$. Recalling previous remarks about the radial acceleration, Newton's second law applied to the horizontal axis at Q ($+x$ leftward) leads to

$$\begin{aligned} N &= m\,\frac{v_{\text{com}}^2}{R-r} \\ &= m\,\frac{50gR}{7(R-r)} \end{aligned}$$

which (for $R \gg r$) gives $N \approx 50mg/7$.